ON INDEFINITE INTEGRAL

(MANUAL FOR CALCULATING ANTIDERIVATIVES AND DEFINITE INTEGRALS IN EXCEL)

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NOTE: This ebook is in letter size

ANTIDERIVATIVES

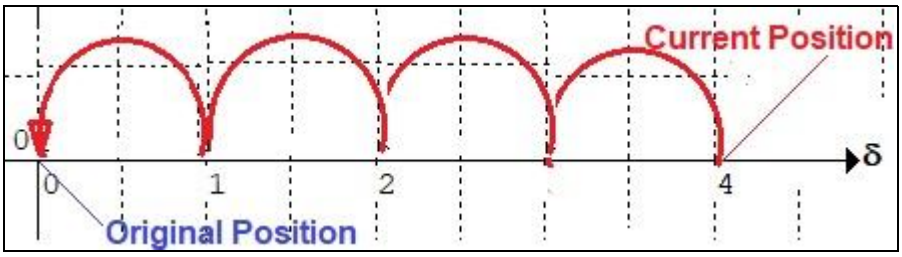
SOLVING ANTIDERIVATIVES WITH EXCEL

Find the Antiderivative of:

$$\frac{d^4}{dx^4} = 720x$$

Initial condition (x,y,y',y'',y''')=(1,14; 36,126,360)

Sol…



We can see that the derivative function is 4 units to the right of the origin (***Original Function***), so we must move 4 units to the left to reach the ***original function***.

To do that, we use the following definition:

$$\rightarrow \frac{d^{(\delta-m)}}{dx^{(\delta-m)}}y = \frac{d^{-m}}{dx^{-m}}f(x) + \sum_{k=1}^{m}\left[C_k.\frac{d^{-(\boldsymbol{m}-k)}}{dx^{-(\boldsymbol{m}-k)}}x^0\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\Omega)$$

Where the ***derivatives with a negative sign*** represent ***integrals***.

We must go one by one to the left until we reach the original function.

NOTE: On the right-hand side of the definition (***Ω***), both the part **NOT** within the brackets [ ], and the part **THAT IS** within the brackets [ ] are obtained from ***Definition (1.1)*** (page 74) of the book "***On Indefinite Integral***". However, the part THAT IS within the brackets [ ] can also be more easily obtained using ***Definition (2.2)*** (page 108) of the aforementioned book.

The following are the four equations that will be used to return the position function ***δ=4*** to it's original position (*δ=0*).

m=1

I) First step to the left, therefore ***m=1.***

$$\rightarrow \frac{d^{(\delta-\boldsymbol{m})}}{dx^{(\delta-\boldsymbol{m})}}y = \frac{d^{-\boldsymbol{m}}}{dx^{-\boldsymbol{m}}}f(x) + \sum_{k=1}^{\boldsymbol{m}}\left[C_k.\frac{d^{-(\boldsymbol{m}-k)}}{dx^{-(\boldsymbol{m}-k)}}x^0\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\Omega)$$

$$\frac{d^{(4-\boldsymbol{1})}}{dx^{(4-\boldsymbol{1})}}y = \frac{d^{-\boldsymbol{1}}}{dx^{-\boldsymbol{1}}}f(x) + \sum_{k=1}^{\boldsymbol{1}}\left[C_k.\frac{d^{-(\boldsymbol{1}-k)}}{dx^{-(\boldsymbol{1}-k)}}x^0\right]$$

$$\frac{d^3}{dx^3}y = \frac{d^{-\boldsymbol{1}}}{dx^{-\boldsymbol{1}}}f(x) + \left[C_1.\frac{d^{-(\boldsymbol{1}-1)}}{dx^{-(\boldsymbol{1}-1)}}x^0\right]$$

$$\frac{d^3}{dx^3}y = \frac{d^{-\boldsymbol{1}}}{dx^{-\boldsymbol{1}}}f(x) + \left[C_1.\frac{d^{-(0)}}{dx^{-(0)}}x^0\right]$$

$$\frac{d^3}{dx^3}y = \frac{d^{-\boldsymbol{1}}}{dx^{-\boldsymbol{1}}}f(x) + \left[C_1.\frac{d^0}{dx^0}x^0\right]$$

$$\frac{d^3}{dx^3}y = \frac{d^{-\boldsymbol{1}}}{dx^{-\boldsymbol{1}}}f(x) + [C_1]$$

$$\frac{d^3}{dx^3}y = \frac{d^{-\boldsymbol{1}}}{dx^{-\boldsymbol{1}}}f(x) + C_1$$

Using **Definition (1.1)** for the right-hand side which still contains a derivative of negative order (which is itself an integral), the last expression becomes:

$$\frac{d^3}{dx^3}y = \int f(x)\,dx + C_1$$

$$\frac{d^3}{dx^3}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n} f\left[x - \left(\frac{x}{n}\right).(k)\right]\right\}.\left(\frac{x}{n}\right)\right\} + C_1$$

However, since Excel will be used to numerically find the ***Original Function***, the last expression above becomes the following (by choosing a very large number ***N*** that tends to infinity):

$$\boldsymbol{f'''(x_0) \cong \left\{\left\{\sum_{k=1}^{N} f\left[x_0 - \left(\frac{x_0}{N}\right).(k)\right]\right\}.\left(\frac{x_0}{N}\right)\right\} + C_1 \ldots\ldots(m1)}$$

m=2

II) Second step to the left, therefore ***m=2.***

$$\rightarrow \frac{d^{(\delta-\boldsymbol{m})}}{dx^{(\delta-\boldsymbol{m})}}y = \frac{d^{-\boldsymbol{m}}}{dx^{-\boldsymbol{m}}}f(x) + \sum_{k=1}^{\boldsymbol{m}}\left[C_k.\frac{d^{-(\boldsymbol{m}-k)}}{dx^{-(\boldsymbol{m}-k)}}x^0\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\Omega)$$

$$\frac{d^{(4-\boldsymbol{2})}}{dx^{(4-\boldsymbol{2})}}y = \frac{d^{-\boldsymbol{2}}}{dx^{-\boldsymbol{2}}}f(x) + \sum_{k=1}^{\boldsymbol{2}}\left[C_k.\frac{d^{-(\boldsymbol{2}-k)}}{dx^{-(\boldsymbol{2}-k)}}x^0\right]$$

$$\frac{d^2}{dx^2}y = \frac{d^{-\boldsymbol{2}}}{dx^{-\boldsymbol{2}}}f(x) + \left[C_1.\frac{d^{-(\boldsymbol{2}-1)}}{dx^{-(\boldsymbol{2}-1)}}x^0 + C_2.\frac{d^{-(\boldsymbol{2}-2)}}{dx^{-(\boldsymbol{2}-2)}}x^0\right]$$

$$\frac{d^2}{dx^2}y = \frac{d^{-\boldsymbol{2}}}{dx^{-\boldsymbol{2}}}f(x) + \left[C_1.\frac{d^{-1}}{dx^{-1}}x^0 + C_2.\frac{d^{-(\boldsymbol{0})}}{dx^{-(0)}}x^0\right]$$

$$\frac{d^2}{dx^2}y = \frac{d^{-\boldsymbol{2}}}{dx^{-\boldsymbol{2}}}f(x) + \left[C_1.\frac{d^{-1}}{dx^{-1}}x^0 + C_2.\frac{d^0}{dx^0}x^0\right]$$

$$\frac{d^2}{dx^2}y = \frac{d^{-\boldsymbol{2}}}{dx^{-\boldsymbol{2}}}f(x) + [C_1x + C_2]$$

$$\frac{d^2}{dx^2}y = \frac{d^{-\boldsymbol{2}}}{dx^{-\boldsymbol{2}}}f(x) + C_1x + C_2$$

Using **Definition (1.1)** for the right-hand side which still contains a derivative of negative order (which is itself an integral), the last expression becomes:

$$\frac{d^2}{dx^2}y = \int\int f(x)\,dx^2 + C_1x + C_2$$

$$\frac{d^2}{dx^2}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-1}k.f\left[x - \left(\frac{x}{n}\right).(k+1)\right]\right\}.\left(\frac{x}{n}\right)^2\right\} + C_1x + C_2$$

However, since Excel will be used to numerically find the ***Original Function***, the last expression above becomes the following (by choosing a very large number ***N*** that tends to infinity):

$$\boldsymbol{f''(x_0) \cong \left\{\left\{\sum_{k=1}^{N-1}k.f\left[x_0 - \left(\frac{x_0}{N}\right).(k+1)\right]\right\}.\left(\frac{x_0}{N}\right)^2\right\} + C_1x + C_2 \ldots\ldots(m2)}$$

m=3

III) Third step to the left, so ***m=3.***

$$\rightarrow \frac{d^{(\delta-\boldsymbol{m})}}{dx^{(\delta-\boldsymbol{m})}}y = \frac{d^{-\boldsymbol{m}}}{dx^{-\boldsymbol{m}}}f(x) + \sum_{k=1}^{\boldsymbol{m}}\left[C_k.\frac{d^{-(\boldsymbol{m}-k)}}{dx^{-(\boldsymbol{m}-k)}}x^0\right] \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(\Omega)$$

$$\frac{d^{(4-\boldsymbol{3})}}{dx^{(4-\boldsymbol{3})}}y = \frac{d^{-\boldsymbol{3}}}{dx^{-\boldsymbol{3}}}f(x) + \sum_{k=1}^{\boldsymbol{3}}\left[C_k.\frac{d^{-(\boldsymbol{3}-k)}}{dx^{-(\boldsymbol{3}-k)}}x^0\right]$$

$$\frac{d^1}{dx^1}y = \frac{d^{-\boldsymbol{3}}}{dx^{-\boldsymbol{3}}}f(x) + \left[C_1.\frac{d^{-(\boldsymbol{3}-1)}}{dx^{-(\boldsymbol{3}-1)}}x^0 + C_2.\frac{d^{-(\boldsymbol{3}-2)}}{dx^{-(\boldsymbol{3}-2)}}x^0 + C_3.\frac{d^{-(\boldsymbol{3}-3)}}{dx^{-(\boldsymbol{3}-3)}}x^0\right]$$

$$\frac{d^1}{dx^1}y = \frac{d^{-\boldsymbol{3}}}{dx^{-\boldsymbol{3}}}f(x) + \left[C_1.\frac{d^{-2}}{dx^{-2}}x^0 + C_2.\frac{d^{-1}}{dx^{-1}}x^0 + C_3.\frac{d^{-(0)}}{dx^{-(0)}}x^0\right]$$

$$\frac{d^1}{dx^1}y = \frac{d^{-\boldsymbol{3}}}{dx^{-\boldsymbol{3}}}f(x) + \left[C_1.\frac{d^{-2}}{dx^{-2}}x^0 + C_2.\frac{d^{-1}}{dx^{-1}}x^0 + C_3.\frac{d^0}{dx^0}x^0\right]$$

$$\frac{d^1}{dx^1}y = \frac{d^{-\boldsymbol{3}}}{dx^{-\boldsymbol{3}}}f(x) + \left[\frac{1}{2}C_1x^2 + C_2x + C_3\right]$$

$$\frac{d^1}{dx^1}y = \frac{d^{-\boldsymbol{3}}}{dx^{-\boldsymbol{3}}}f(x) + \frac{1}{2}C_1x^2 + C_2x + C_3$$

Using **Definition (1.1)** for the right-hand side which still contains a derivative of negative order (which is itself an integral), the last expression becomes:

$$\frac{d^1}{dx^1}y = \int\int\int f(x)\,dx^3 + \frac{1}{2}C_1x^2 + C_2x + C_3$$

$$\frac{d^1}{dx^1}y = \left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-2}\left(\frac{k^2+k}{2}\right).f\left[x-\left(\frac{x}{n}\right).(k+2)\right]\right\}.\left(\frac{x}{n}\right)^3\right\} + \frac{1}{2}C_1x^2 + C_2x + C_3$$

However, since Excel will be used to numerically find the ***Original Function***, the last expression above becomes the following (by choosing a very large number ***N*** that tends to infinity):

$$\boldsymbol{f^{'}(x_0) \cong \left\{\left\{\sum_{k=1}^{N-2}\left(\frac{k^2+k}{2}\right).f\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]\right\}.\left(\frac{x_0}{N}\right)^3\right\} + \frac{1}{2}C_1x^2 + C_2x + C_3 \cdots\cdots(m3)}$$

m=4

IV) Fourth step to the left, so ***m=4.***

$$\rightarrow \frac{d^{(\delta-\boldsymbol{m})}}{dx^{(\delta-\boldsymbol{m})}}y=\frac{d^{-\boldsymbol{m}}}{dx^{-\boldsymbol{m}}}f(x)+\sum_{k=1}^{\boldsymbol{m}}\left[C_k.\frac{d^{-(\boldsymbol{m}-k)}}{dx^{-(\boldsymbol{m}-k)}}x^0\right]\cdots\cdots\cdots\cdots\cdots\cdots\cdots(\Omega)$$

$$\frac{d^{(4-\boldsymbol{4})}}{dx^{(4-\boldsymbol{4})}}y=\frac{d^{-\boldsymbol{4}}}{dx^{-\boldsymbol{4}}}f(x)+\sum_{k=1}^{\boldsymbol{4}}\left[C_k.\frac{d^{-(\boldsymbol{4}-k)}}{dx^{-(\boldsymbol{4}-k)}}x^0\right]$$

$$\frac{d^0}{dx^0}y=\frac{d^{-\boldsymbol{4}}}{dx^{-\boldsymbol{4}}}f(x)+\left[C_1.\frac{d^{-(\boldsymbol{4}-1)}}{dx^{-(\boldsymbol{4}-1)}}x^0+C_2.\frac{d^{-(\boldsymbol{4}-2)}}{dx^{-(\boldsymbol{4}-2)}}x^0+C_3.\frac{d^{-(\boldsymbol{4}-3)}}{dx^{-(\boldsymbol{4}-3)}}x^0+C_4.\frac{d^{-(\boldsymbol{4}-4)}}{dx^{-(\boldsymbol{4}-4)}}x^0\right]$$

$$y=\frac{d^{-\boldsymbol{4}}}{dx^{-\boldsymbol{4}}}f(x)+\left[C_1.\frac{d^{-3}}{dx^{-3}}x^0+C_2.\frac{d^{-2}}{dx^{-2}}x^0+C_3.\frac{d^{-1}}{dx^{-1}}x^0+C_4.\frac{d^{-(0)}}{dx^{-(0)}}x^0\right]$$

$$y=\frac{d^{-\boldsymbol{4}}}{dx^{-\boldsymbol{4}}}f(x)+\left[C_1.\frac{d^{-3}}{dx^{-3}}x^0+C_2.\frac{d^{-2}}{dx^{-2}}x^0+C_3.\frac{d^{-1}}{dx^{-1}}x^0+C_4.\frac{d^{0}}{dx^{0}}x^0\right]$$

$$y=\frac{d^{-\boldsymbol{4}}}{dx^{-\boldsymbol{4}}}f(x)+\left[\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4\right]$$

$$y=\frac{d^{-\boldsymbol{4}}}{dx^{-\boldsymbol{4}}}f(x)+\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4$$

Using **Definition (1.1)** for the right-hand side which still contains a derivative of negative order (which is itself an integral), the last expression becomes:

$$y=\int\int\int\int f(x)\,dx^4+\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4$$

$$y=\left\{\lim_{n\to\infty}\left\{\sum_{k=1}^{n-3}\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x-\left(\frac{x}{n}\right).(k+3)\right]\right\}.\left(\frac{x}{n}\right)^4\right\}+\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4$$

However, since Excel will be used to numerically find the ***Original Function***, the last expression above becomes the following (by choosing a very large number ***N*** that tends to infinity):

$$f(x_0)\cong\left\{\left\{\sum_{k=1}^{N-3}\left[\frac{(\boldsymbol{k})(\boldsymbol{k}+1)(\boldsymbol{k}+2)}{\boldsymbol{6}}\right].f\left[x_0-\left(\frac{x_0}{N}\right).(\boldsymbol{k}+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4\cdots\cdots(m4)$$

The 4 equations that will allow you to return to the Original Function (in Excel) are, therefore:

$$f'''(x_0) \cong \left\{\left\{\sum_{k=1}^{N} f\left[x_0 - \left(\frac{x_0}{N}\right).(k)\right]\right\}.\left(\frac{x_0}{N}\right)\right\} + C_1 \quad \cdots\cdots(m1)$$

$$f''(x_0) \cong \left\{\left\{\sum_{k=1}^{N-1} k.f\left[x_0 - \left(\frac{x_0}{N}\right).(k+1)\right]\right\}.\left(\frac{x_0}{N}\right)^2\right\} + (C_1)(x_0) + C_2 \quad \cdots\cdots(m2)$$

$$f'(x_0) \cong \left\{\left\{\sum_{k=1}^{N-2} \left(\frac{k^2+k}{2}\right).f\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]\right\}.\left(\frac{x_0}{N}\right)^3\right\} + \frac{1}{2}(C_1)(x_0)^2 + (C_2)(x_0) + C_3 \quad \cdots\cdots(m3)$$

$$f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3} \left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\} + \frac{1}{6}(C_1)(x_0)^3 + \frac{1}{2}(C_2)(x_0)^2 + (C_3)(x_0) + C_4 \quad \cdots\cdots(m4)$$

Since:

$$\rightarrow \frac{d^4}{dx^4} = 720x$$

For the point (x,y,y',y'',y''')=(1,14; 36,126,360)

For ***N=100,000*** (an approximation to "***infinity***")

To simplify calculations in Excel, the four equations above can also be written as:

$$f'''(x_0) \cong \left\{\sum_{k=1}^{N} \left\{f\left[x_0 - \left(\frac{x_0}{N}\right).(k)\right].\left(\frac{x_0}{N}\right)\right\}\right\} + C_1 \quad \cdots\cdots(m1)$$

$$f''(x_0) \cong \left\{\sum_{k=1}^{N-1} \left\{k.f\left[x_0 - \left(\frac{x_0}{N}\right).(k+1)\right].\left(\frac{x_0}{N}\right)^2\right\}\right\} + (C_1)(x_0) + C_2 \quad \cdots\cdots(m2)$$

$$f'(x_0) \cong \left\{\sum_{k=1}^{N-2} \left\{\left(\frac{k^2+k}{2}\right).f\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}\right\} + \frac{1}{2}(C_1)(x_0)^2 + (C_2)(x_0) + C_3 \quad \cdots\cdots(m3)$$

$$f(x_0) \cong \left\{\sum_{k=1}^{N-3} \left\{\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}\right\} + \frac{1}{6}(C_1)(x_0)^3 + \frac{1}{2}(C_2)(x_0)^2 + (C_3)(x_0) + C_4 \quad \cdots\cdots(m4)$$

NOTE: If ***f(x)=720x***, you have to replace that ***x*** with $\left[x - \left(\frac{x}{n}\right).k\right]$, so that it looks like this:

$$f\left[x - \left(\frac{x}{n}\right).k\right] = 720\left[x - \left(\frac{x}{n}\right).k\right] \equiv f\left[x_0 - \left(\frac{x_0}{n}\right).k\right] = 720\left[x_0 - \left(\frac{x_0}{n}\right).k\right]$$

By substituting ***f(x)***, the initial conditions and ***N=100,000***, the four previous equations become:

$$360 \cong \left\{ \sum_{k=1}^{100{,}000} \left\{ 720 \left[1 - \left(\frac{1}{100{,}000} \right) . (k) \right] . \left(\frac{1}{100{,}000} \right) \right\} \right\} + C_1 \quad \cdots\cdots (m1)$$

$$126 \cong \left\{ \sum_{k=1}^{100{,}000-1} \left\{ k . 720 \left[1 - \left(\frac{1}{100{,}000} \right) . (k+1) \right] . \left(\frac{1}{100{,}000} \right)^2 \right\} \right\} + (C_1)(1) + C_2 \quad \cdots\cdots (m2)$$

$$36 \cong \left\{ \sum_{k=1}^{100{,}000-2} \left\{ \left(\frac{k^2+k}{2} \right) . 720 \left[1 - \left(\frac{1}{100{,}000} \right) . (k+2) \right] . \left(\frac{1}{100{,}000} \right)^3 \right\} \right\} + \frac{1}{2}(C_1)(1)^2 + (C_2)(1) + C_3 \quad \cdots\cdots (m3)$$

$$14 \cong \left\{ \sum_{k=1}^{100{,}000-3} \left\{ \left[\frac{(k)(k+1)(k+2)}{6} \right] . 720 \left[1 - \left(\frac{1}{100{,}000} \right) . (k+3) \right] . \left(\frac{1}{100{,}000} \right)^4 \right\} \right\} + \frac{1}{6}(C_1)(1)^3 + \frac{1}{2}(C_2)(1)^2 + (C_3)(1) + C_4 \quad \cdots\cdots (m4)$$

Solving the four equations, we finally get:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100{,}000} \left\{ 720 \left[1 - \left(\frac{1}{100{,}000} \right) . (k) \right] . \left(\frac{1}{100{,}000} \right) \right\} \right\} \quad \cdots\cdots (M1)$$

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99{,}999} \left\{ 720k \left[1 - \left(\frac{1}{100{,}000} \right) . (k+1) \right] . \left(\frac{1}{100{,}000} \right)^2 \right\} \right\} - (C_1) \quad \cdots\cdots (M2)$$

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99{,}998} \left\{ 720 \left(\frac{k^2+k}{2} \right) \left[1 - \left(\frac{1}{100{,}000} \right) . (k+2) \right] . \left(\frac{1}{100{,}000} \right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \quad \cdots\cdots (M3)$$

$$C_4 \cong 14 - \left\{ \sum_{k=1}^{99{,}997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100{,}000} \right) . (k+3) \right] . \left(\frac{1}{100{,}000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \quad \cdots\cdots (M4)$$

CALCULATING M=1 IN EXCEL

We open an Excel sheet and name it ***M=1***

The equation ***(M1)*** is:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$$

1) We write this equation in **cell A1**:

2) In **cell A10**, we begin to write the following:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20				
21				

3) In **cell A20**, we begin to put the following:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21				
22				
23				

According:

$$C_1 \cong f'''(x_0) - \left\{ \sum_{k=1}^{N} \left\{ f \left[x_0 - \left(\frac{x_0}{N} \right) . (k) \right] . \left(\frac{x_0}{N} \right) \right\} \right\} \text{ ……} (m1)$$

$\left[x_0 - \left(\frac{x_0}{N} \right) . (k) \right]$ Represent the **modified *x*** to *enter it in* **(M1)**.

$f \left[x_0 - \left(\frac{x_0}{N} \right) . (k) \right]$ Represents that ***modified x*** substituted in the function ***f(x).***

4) According to the equation **(M1)**, ***k*** must range from ***k=1*** to ***k= 100,000*** since the ***N*** chosen for this case is equal to ***N=100,000*** (a large number to be considered a tendency towards infinity (∞); however, a much larger ***N*** such as ***N=1,000,000*** or more could be chosen). Therefore, we place in **cell A21** and write the following:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21				
22				
23				

=SEQUENCE(100000,1,1,1)

This means there will be **100,000 Rows**
In **1 Column**
Starting **from 1**
Increasing by 1 (from 1 until reaching N=100,000)

Therefore, it will be as follows:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			
100020	100,000			

5) Now in **cell B21** we write the following:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			
100020	100,000			

=(B10)-((B12)*(A21))

This part corresponds to equation (M1) in red:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \quad \ldots\ldots (M1)$$

6) It will look like this (after autofilling up to ***k=100,000***):

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999		
22	2	0.99998		
23	3	0.99997		
24	4	0.99996		
25	5	0.99995		
100016	99,996	0.00004		
100017	99,997	0.00003		
100018	99,998	0.00002		
100019	99,999	0.00001		
100020	100,000	0.00000		

7) In **cell C21**, we write the following:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999		
22	2	0.99998		
23	3	0.99997		
24	4	0.99996		
25	5	0.99995		
100016	99,996	0.00004		
100017	99,997	0.00003		
100018	99,998	0.00002		
100019	99,999	0.00001		
100020	100,000	0.00000		

=720*B21

This part corresponds to equation (M1) in red:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \text{……} (M1)$$

8) It will look like this (after autofilling up to ***k=100,000***):

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	
22	2	0.99998	719.98560	
23	3	0.99997	719.97840	
24	4	0.99996	719.97120	
25	5	0.99995	719.96400	
100016	99,996	0.00004	0.02880	
100017	99,997	0.00003	0.02160	
100018	99,998	0.00002	0.01440	
100019	99,999	0.00001	0.00720	
100020	100,000	0.00000	0.00000	

9) In **cell D21**, we write the following:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	
22	2	0.99998	719.98560	
23	3	0.99997	719.97840	
24	4	0.99996	719.97120	
25	5	0.99995	719.96400	
100016	99,996	0.00004	0.02880	
100017	99,997	0.00003	0.02160	
100018	99,998	0.00002	0.01440	
100019	99,999	0.00001	0.00720	
100020	100,000	0.00000	0.00000	

=C21*(B12)

This part corresponds to equation (M1) in red:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$$

10) It will look like this (after autofilling up to ***k=100,000***):

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021				
100022				
100023				
100024				

11) In **cell C100021**, we write **"SUM"**:

12) In **cell D100021**, we write:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUM	
100022				
100023				
100024				

=SUM(D21:D100020)

This part corresponds to equation (M1) in red:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \text{……}(M1)$$

13) It will look like this:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUM	360
100022				
100023				
100024				
100025				

14) In **cell C100023**, we write:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUM	360
100022				
100023			f'''(x_0)	=B13
100024			(-) SUMMATION	=D100021
100025			(=) C1	=B13-D100021

According:

$$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\} \cdots\cdots (M1)$$

Finally:

	A	B	C	D
1	$C_1 \cong 360 - \left\{ \sum_{k=1}^{100,000} \left\{ 720 \left[1 - \left(\frac{1}{100,000} \right) . (k) \right] . \left(\frac{1}{100,000} \right) \right\} \right\}$ ……$(M1)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'''(x_0)=	360		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right]$	$\left\{ 720 \left[x_0 - \left(\frac{x_0}{N} \right) k \right] \right\} \left(\frac{x_0}{N} \right)$
21	1	0.99999	719.99280	0.0071999
22	2	0.99998	719.98560	0.0071999
23	3	0.99997	719.97840	0.0071998
24	4	0.99996	719.97120	0.0071997
25	5	0.99995	719.96400	0.0071996
100016	99,996	0.00004	0.02880	0.0000003
100017	99,997	0.00003	0.02160	0.0000002
100018	99,998	0.00002	0.01440	0.0000001
100019	99,999	0.00001	0.00720	0.0000001
100020	100,000	0.00000	0.00000	0.0000000
100021			SUM	360
100022				
100023			f'''(x_0)	360
100024		(-)	SUMMATION	360
100025		(=)	C1	0

$\therefore C_1 = 0$

CALCULATING M=2 IN EXCEL

We open another Excel sheet and name it ***M=2***

The equation ***(M2)*** is:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

1) We write this equation in **cell A1**:

2) In **cell A10**, we begin to write the following:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20				
21				

3) In **cell A20**, we begin to put the following:

	A	B	C	D
1	$C_2 \cong 126 - \left\{\sum_{k=1}^{99,999}\left\{720k\left[1-\left(\frac{1}{100,000}\right).(k+1)\right].\left(\frac{1}{100,000}\right)^2\right\}\right\} - (C_1)$ ……$(M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+1)\right]$	$720k\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right]$	$\left\{720k\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right].\left(\frac{x_0}{N}\right)^2\right\}$
21				
22				
23				

According:

$$C_2 \cong f''(x_0) - \left\{\sum_{k=1}^{N-1}\left\{k.f\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right].\left(\frac{x_0}{N}\right)^2\right\}\right\} - (C_1)(x_0) \text{ ……} (m2)$$

$\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right]$ Represent the **modified *x*** to *enter it in* **(M2)**.

$f\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right]$ Represents that ***modified x*** substituted in the function ***f(x).***

4) According to the equation **(M2)**, ***k*** must range from ***k=1*** to ***k= 99,999*** since the ***N*** chosen for this case is equal to ***N=100,000*** (a large number to be considered a tendency towards infinity (∞); however, a much larger ***N*** such as ***N=1,000,000*** or more could be chosen). Therefore, we place in **cell A21** and write the following:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21				
22				
23				

=SEQUENCE(100000-1,1,1,1)

This means there will be **99,999 Rows**
In **1 Column**
Starting **from 1**
Increasing by 1 (from 1 until reaching N-1=99,999)

Therefore, it will be as follows:

	A	B	C	D
1	$C_2 \cong 126 - \left\{\sum_{k=1}^{99,999}\left\{720k\left[1-\left(\frac{1}{100,000}\right).(k+1)\right].\left(\frac{1}{100,000}\right)^2\right\}\right\} - (C_1)$ ……$(M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+1)\right]$	$720k\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right]$	$\left\{720k\left[x_0-\left(\frac{x_0}{N}\right).(k+1)\right].\left(\frac{x_0}{N}\right)^2\right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			

5) Now in **cell B21** we write the following:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			
100019	99,999			

=(B10)-((B12)*(A21+1))

This part corresponds to equation (M2) in red:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

6) It will look like this (after autofilling up to ***k=99,999***):

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998		
22	2	0.99997		
23	3	0.99996		
24	4	0.99995		
25	5	0.99994		
100016	99,996	0.00003		
100017	99,997	0.00002		
100018	99,998	0.00001		
100019	99,999	0.00000		

7) In **cell C21**, we write the following:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998		
22	2	0.99997		
23	3	0.99996		
24	4	0.99995		
25	5	0.99994		
100016	99,996	0.00003		
100017	99,997	0.00002		
100018	99,998	0.00001		
100019	99,999	0.00000		

=720*A21*B21

This part corresponds to equation (M2) in red:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

8) It will look like this (after autofilling up to ***k=99,999***):

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	
22	2	0.99997	1,439.95680	
23	3	0.99996	2,159.91360	
24	4	0.99995	2,879.85600	
25	5	0.99994	3,599.78400	
100016	99,996	0.00003	2,159.91360	
100017	99,997	0.00002	1,439.95680	
100018	99,998	0.00001	719.98560	
100019	99,999	0.00000	0.00000	

9) In **cell D21**, we write the following:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	
22	2	0.99997	1,439.95680	
23	3	0.99996	2,159.91360	
24	4	0.99995	2,879.85600	
25	5	0.99994	3,599.78400	
100016	99,996	0.00003	2,159.91360	
100017	99,997	0.00002	1,439.95680	
100018	99,998	0.00001	719.98560	
100019	99,999	0.00000	0.00000	

=C21*(B12)^2

This part corresponds to equation (M2) in red:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

10) It will look like this (after autofilling up to ***k=99,999***):

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000

11) In **cell C100021**, we write **"SUM"**:

12) In **cell D100021**, we write:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100020				
100021			SUM	
100022				
100023				
100024				

=SUM(D21:D100019)

This part corresponds to equation (M2) in red:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

13) It will look like this:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100021			SUM	119.9964

14) In **cell C100023**, we write:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100020				
100021			SUM	119.9964
100022				
100023			f''(x_0)	=B13
100024			(-) SUMMATION	=D100021
100025			(-) C1(previously calculated)	0
100026			(=) C2	=B13-D100021-D100025

According:

$$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$$

Finally:

	A	B	C	D
1	$C_2 \cong 126 - \left\{ \sum_{k=1}^{99,999} \left\{ 720k \left[1 - \left(\frac{1}{100,000} \right) . (k+1) \right] . \left(\frac{1}{100,000} \right)^2 \right\} \right\} - (C_1) \cdots\cdots (M2)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f''(x_0)=	126		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+1) \right]$	$720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right]$	$\left\{ 720k \left[x_0 - \left(\frac{x_0}{N} \right) . (k+1) \right] . \left(\frac{x_0}{N} \right)^2 \right\}$
21	1	0.99998	719.98560	0.00000007
22	2	0.99997	1,439.95680	0.00000014
23	3	0.99996	2,159.91360	0.00000022
24	4	0.99995	2,879.85600	0.00000029
25	5	0.99994	3,599.78400	0.00000036
100016	99,996	0.00003	2,159.91360	0.00000022
100017	99,997	0.00002	1,439.95680	0.00000014
100018	99,998	0.00001	719.98560	0.00000007
100019	99,999	0.00000	0.00000	0.00000000
100020				
100021			SUM	119.9964
100022				
100023			f''(x_0)	126
100024		(-)	SUMMATION	119.9964
100025		(-)	C1(previously calculated)	0
100026		(=)	C2	6.0036

$\therefore C_2 = 6.0036$

CALCULATING M=3 IN EXCEL

We open another Excel sheet and name it ***M=3***

The equation ***(M3)*** is:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720 \left(\frac{k^2+k}{2} \right) \left[1 - \left(\frac{1}{100,000} \right) . (k+2) \right] . \left(\frac{1}{100,000} \right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

1) We write this equation in **cell A1**:

2) In **cell A10**, we begin to write the following:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720 \left(\frac{k^2+k}{2} \right) \left[1 - \left(\frac{1}{100,000} \right) . (k+2) \right] . \left(\frac{1}{100,000} \right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20				
21				

3) In **cell A20**, we begin to put the following:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……(M3)			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21				
22				
23				

According:

$$C_3 \cong f'(x_0) - \left\{ \sum_{k=1}^{N-2} \left\{ \left(\frac{k^2+k}{2}\right).f\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3 \right\} \right\} - \frac{1}{2}(C_1)(x_0)^2 - (C_2)(x_0)) \text{ ……(m3)}$$

$\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]$ Represent the **modified *x*** to *enter it in* **(M3)**.

$f\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]$ Represents that ***modified x*** substituted in the function ***f(x).***

4) According to the equation **(M3)**, ***k*** must range from ***k=1*** to ***k= 99,998*** since the ***N*** chosen for this case is equal to ***N=100,000*** (a large number to be considered a tendency towards infinity (∞); however, a much larger ***N*** such as ***N=1,000,000*** or more could be chosen). Therefore, we place in **cell A21** and write the following:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21				
22				
23				

=SEQUENCE(100000-2,1,1,1)

This means there will be **99,998 Rows**
In **1 Column**
Starting **from 1**
Increasing by 1 (from 1 until reaching N-2=99,998)

Therefore, it will be as follows:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			

5) Now in **cell B21** we write the following:

	A	B	C	D
1	$C_3 \cong 36 - \left\{\sum_{k=1}^{99,998}\left\{720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3\right\}\right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			
100018	99,998			

=(B10)-((B12)*(A21+2))

This part corresponds to equation (M3) in red:

$$C_3 \cong 36 - \left\{\sum_{k=1}^{99,998}\left\{720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3\right\}\right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

6) It will look like this (after autofilling up to ***k=99,998***):

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99{,}998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100{,}000}\right)\cdot(k+2)\right]\cdot\left(\frac{1}{100{,}000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right)\cdot(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right)\cdot(k+2)\right]\cdot\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997		
22	2	0.99996		
23	3	0.99995		
24	4	0.99994		
25	5	0.99993		
100016	99,996	0.00002		
100017	99,997	0.00001		
100018	99,998	0.00000		

7) In **cell C21**, we write the following:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997		
22	2	0.99996		
23	3	0.99995		
24	4	0.99994		
25	5	0.99993		
100016	99,996	0.00002		
100017	99,997	0.00001		
100018	99,998	0.00000		

=720*(((A21^2+A21)/2))*B21

This part corresponds to equation (M3) in red:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

8) It will look like this (after autofilling up to ***k=99,998***):

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99{,}998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100{,}000}\right).(k+2)\right].\left(\frac{1}{100{,}000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	
22	2	0.99996	2,159.91360	
23	3	0.99995	4,319.78400	
24	4	0.99994	7,199.56800	
25	5	0.99993	10,799.24400	
100016	99,996	0.00002	71,994,960.08607	
100017	99,997	0.00001	35,998,200.02144	
100018	99,998	0.00000	0.00000	

9) In **cell D21**, we write the following:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	
22	2	0.99996	2,159.91360	
23	3	0.99995	4,319.78400	
24	4	0.99994	7,199.56800	
25	5	0.99993	10,799.24400	
100016	99,996	0.00002	71,994,960.08607	
100017	99,997	0.00001	35,998,200.02144	
100018	99,998	0.00000	0.00000	

=C21*(B12)^3

This part corresponds to equation (M3) in red:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

10) It will look like this (after autofilling up to ***k=99,998***):

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99{,}998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100{,}000}\right).(k+2)\right].\left(\frac{1}{100{,}000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000

11) In **cell C100021**, we write **"SUM"**:

12) In **cell D100021**, we write:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100019				
100020				
100021			SUM	
100022				
100023				
100024				

=SUM(D21:D100018)

This part corresponds to equation (M3) in red:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

13) It will look like this:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right)\cdot(k+2)\right]\cdot\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right)\cdot(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right)\cdot(k+2)\right]\cdot\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100021			SUM	29.9982

14) In **cell C100023**, we write:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100021			**SUM**	**29.9982**
100022				
100023			**f'(x_0)**	=B13
100024		**(-)**	**SUMMATION**	=D100021
100025		**(-)**	**(1/2)C1**(previously calculated)	**=(1/2)*0**
100026		**(-)**	**C2**(previously calculated)	**6.0036**
100027		**(=)**	**C3**	=B13-D100021-D100025-D100026

According:

$$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2) \cdots\cdots (M3)$$

Finally:

	A	B	C	D
1	$C_3 \cong 36 - \left\{ \sum_{k=1}^{99,998} \left\{ 720\left(\frac{k^2+k}{2}\right)\left[1-\left(\frac{1}{100,000}\right).(k+2)\right].\left(\frac{1}{100,000}\right)^3 \right\} \right\} - \frac{1}{2}(C_1) - (C_2)$ ……$(M3)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f'(x_0)=	36		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+2)\right]$	$720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99997	719.97840	0.00000
22	2	0.99996	2,159.91360	0.00000
23	3	0.99995	4,319.78400	0.00000
24	4	0.99994	7,199.56800	0.00000
25	5	0.99993	10,799.24400	0.00000
100016	99,996	0.00002	71,994,960.08607	0.00000
100017	99,997	0.00001	35,998,200.02144	0.00000
100018	99,998	0.00000	0.00000	0.00000
100021			SUM	29.9982
100022				
100023			f'(x_0)	36
100024		(-)	SUMMATION	29.9982
100025		(-)	(1/2)C1(previously calculated)	0
100026		(-)	C2(previously calculated)	6.0036
100027		(=)	C3	-0.0018

$$\therefore C_3 = -0.0018$$

CALCULATING M=4 IN EXCEL

We open another Excel sheet and name it ***M=4***

The equation ***(M4)*** is:

$$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) . (k+3) \right] . \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

1) We write this equation in **cell A1**:

2) In **cell A10**, we begin to write the following:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) . (k+3) \right] . \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	=(B10/B11)		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20				
21				

3) In **cell A20**, we begin to put the following:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) . (k+3) \right] . \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……(M4)			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) . (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) . (k+3) \right] . \left(\frac{x_0}{N} \right)^4 \right\}$
21				
22				
23				

According:

$$C_4 \cong f(x_0) - \left\{ \sum_{k=1}^{N-3} \left\{ \left[\frac{(k)(k+1)(k+2)}{6} \right] . f\left[x_0 - \left(\frac{x_0}{N} \right) . (k+3) \right] . \left(\frac{x_0}{N} \right)^4 \right\} \right\} - \frac{1}{6}(C_1)(x_0)^3 - \frac{1}{2}(C_2)(x_0)^2 - (C_3)(x_0) \text{ ……} (m4)$$

$\left[x_0 - \left(\frac{x_0}{N} \right) . (k+3) \right]$ Represent the **modified *x*** to *enter it in* **(M4)**.

$f\left[x_0 - \left(\frac{x_0}{N} \right) . (k+3) \right]$ Represents that ***modified x*** substituted in the function ***f(x).***

4) According to the equation **(M4)**, ***k*** must range from ***k=1*** to ***k= 99,997*** since the ***N*** chosen for this case is equal to ***N=100,000*** (a large number to be considered a tendency towards infinity (∞); however, a much larger ***N*** such as ***N=1,000,000*** or more could be chosen). Therefore, we place in **cell A21** and write the following:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99{,}997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100{,}000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100{,}000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right] \cdot \left(\frac{x_0}{N} \right)^4 \right\}$
21				
22				
23				

=SEQUENCE(100000-3,1,1,1)

This means there will be **99,997 Rows**
In **1 Column**
Starting **from 1**
Increasing by 1 (from 1 until reaching N-3=99,997)

Therefore, it will be as follows:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			

5) Now in **cell B21** we write the following:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1			
22	2			
23	3			
24	4			
25	5			
100016	99,996			
100017	99,997			

=(B10)-((B12)*(A21+3))

This part corresponds to equation (M4) in red:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

6) It will look like this (after autofilling up to ***k=99,997***):

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1	0.99996		
22	2	0.99995		
23	3	0.99994		
24	4	0.99993		
25	5	0.99992		
100016	99,996	0.00001		
100017	99,997	0.00000		

7) In **cell C21**, we write the following:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1	0.99996		
22	2	0.99995		
23	3	0.99994		
24	4	0.99993		
25	5	0.99992		
100016	99,996	0.00001		
100017	99,997	0.00000		

=720*((((A21)*(A21+1)*(A21+2))/6))*B21

This part corresponds to equation (M4) in red:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

8) It will look like this (after autofilling up to ***k=99,997***):

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right].\left(\frac{x_0}{N}\right)^4\right\}$
21	1	0.99996	719.97120	
22	2	0.99995	2,879.85600	
23	3	0.99994	7,199.56800	
24	4	0.99993	14,398.99200	
25	5	0.99992	25,197.98400	
100016	99,996	0.00001	1,199,892,003,114.51000	
100017	99,997	0.00000	0.00000	

9) In **cell D21**, we write the following:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right] \cdot \left(\frac{x_0}{N} \right)^4 \right\}$
21	1	0.99996	719.97120	
22	2	0.99995	2,879.85600	
23	3	0.99994	7,199.56800	
24	4	0.99993	14,398.99200	
25	5	0.99992	25,197.98400	
100016	99,996	0.00001	1,199,892,003,114.51000	
100017	99,997	0.00000	0.00000	

=C21*(B12)^4

This part corresponds to equation (M4) in red:

$$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

10) It will look like this (after autofilling up to ***k=99,997***):

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99{,}997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100{,}000}\right).(k+3)\right].\left(\frac{1}{100{,}000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000

11) In **cell C100021**, we write **"SUM"**:

12) In **cell D100021**, we write:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……(M4)			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100018				
100019				
100020				
100021			SUM	
100022				
100023				
100024				

=SUM(D21:D100017)

This part corresponds to equation (M4) in red:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$$

13) It will look like this:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99{,}997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100{,}000} \right) \cdot (k+3) \right] \cdot \left(\frac{1}{100{,}000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+3) \right]$	$\left\{ 720 \left(\frac{k^2+k}{2} \right) \left[x_0 - \left(\frac{x_0}{N} \right) \cdot (k+2) \right] \cdot \left(\frac{x_0}{N} \right)^3 \right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100018				
100021			SUM	5.9994

14) In **cell C100023**, we write:

	A	B	C	D
1	$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3)$ ……$(M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			**SUM**	**5.9994**
100022				
100023			f(x_0)	=B13
100024			(-) **SUMMATION**	=D100021
100025			(-) (1/6)C1(previously calculated)	0
100026			(-) (1/2)C2(previously calculated)	=(1/2)*(6.0036)
100027			(-) C3(previously calculated)	-0.0018
100028			(=) **C4**	=B13-D100021-D100025-D100026-D100027

According:

$$C_4 \cong 14 - \left\{\sum_{k=1}^{99,997}\left\{720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[1-\left(\frac{1}{100,000}\right).(k+3)\right].\left(\frac{1}{100,000}\right)^4\right\}\right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \text{ ……} (M4)$$

Finally:

	A	B	C	D
1	$C_4 \cong 14 - \left\{ \sum_{k=1}^{99,997} \left\{ 720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[1 - \left(\frac{1}{100,000} \right) . (k+3) \right] . \left(\frac{1}{100,000} \right)^4 \right\} \right\} - \frac{1}{6}(C_1) - \frac{1}{2}(C_2) - (C_3) \cdots\cdots (M4)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N} \right) (k+3) \right]$	$720 \left[\frac{(k)(k+1)(k+2)}{6} \right] \left[x_0 - \left(\frac{x_0}{N} \right) . (k+3) \right]$	$\left\{ 720 \left(\frac{k^2+k}{2} \right) \left[x_0 - \left(\frac{x_0}{N} \right) . (k+2) \right] . \left(\frac{x_0}{N} \right)^3 \right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			SUM	5.9994
100022				
100023			f(x_0)	14
100024		(-)	SUMMATION	5.9994
100025		(-)	(1/6)C1(previously calculated)	0
100026		(-)	(1/2)C2(previously calculated)	3.0018
100027		(-)	C3(previously calculated)	-0.0018
100028		(=)	C4	5.0006

$$\therefore C_4 = 5.0006$$

TESTING

The equation ***(m4)*** (**Page 7**) is:

$$f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+\frac{1}{6}C_1x^3+\frac{1}{2}C_2x^2+C_3x+C_4 \cdots\cdots(m4)$$

$$\rightarrow f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+\frac{1}{6}C_1(x_0)^3+\frac{1}{2}C_2(x_0)^2+C_3(x_0)+C_4 \cdots\cdots(m4\omega)$$

Where we previously obtained the value of the Constants:

$\rightarrow C_1 = 0$

$\rightarrow C_2 = 6.0036$

$\rightarrow C_3 = -0.0018$

$\rightarrow C_4 = 5.0006$

We need to round the Constants, so:

$\rightarrow C_1 = 0$

$\rightarrow C_2 = 6$

$\rightarrow C_3 = 0$

$\rightarrow C_4 = 5$

Therefore, the equation ***(m4ω)*** becomes:

$$\rightarrow f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+\frac{1}{6}(0)(x_0)^3+\frac{1}{2}(6)(x_0)^2+(0)(x_0)+5\cdots\cdots(m4\omega)$$

$$f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+3(x_0)^2+5\cdots\cdots(m4\omega)$$

$$\rightarrow f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\}+3(x_0)^2+5\cdots\cdots(m4\omega)$$

To create a tabulation in Excel, we need to open a fifth Excel sheet and name it ***m=4 Graph*** and then copy all the data from sheet ***M=4.***

So:

1) We write the equation ***(m4ω)*** in the **cell A1**:

2) Starting in **cell A10**, we enter the following data:

	A	B	C	D
1	$f(x_0) \cong \left\{\sum_{k=1}^{N-3} 720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\Bigg\} + 3(x_0)^2 + 5\cdots\cdots(m4\omega)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13	f(x_0)=	14		
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0-\left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0-\left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0-\left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			SUM	5.9994
100022				
100023			f(x_0)	14
100024		(-)	SUMMATION	5.9994
100025		(-)	(1/6)C1(previously calculated)	0
100026		(-)	(1/2)C2(previously calculated))	3.0018
100027		(-)	C3(previously calculated)	-0.0018
100028		(=)	C4	5.0006

3) We delete the contents of **cell A13** and **cell B13** and replace the contents at the bottom as follows (from **cell C100021**):

	A	B	C	D
1	$f(x_0) \cong \left\{\sum_{k=1}^{N-3} 720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4 + 3(x_0)^2 + 5\cdots\cdots(m4\omega)$			
2				
3				
4				
5				
6				
7				
8				
9				
10	x_0=	1		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
14				
15				
16				
17				
18				
19				
20	k	$\left[x_0 - \left(\frac{x_0}{N}\right)(k+3)\right]$	$720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]$	$\left\{720\left(\frac{k^2+k}{2}\right)\left[x_0 - \left(\frac{x_0}{N}\right).(k+2)\right].\left(\frac{x_0}{N}\right)^3\right\}$
21	1	0.99996	719.97120	0.000000
22	2	0.99995	2,879.85600	0.000000
23	3	0.99994	7,199.56800	0.000000
24	4	0.99993	14,398.99200	0.000000
25	5	0.99992	25,197.98400	0.000000
100016	99,996	0.00001	1,199,892,003,114.51	0.00000
100017	99,997	0.00000	0.00000	0.00000
100021			SUMMATION	=SUM(D21:D100017)
100022		(+)	$3(x_0)^2$	=3*(B10)^2
100023		(+)	5	=5
100024		(=)	$f(x_0)$	=D100021+D100022+D100023
100025				
100026				
100027				

According:

$$f(x_0) \cong \left\{\sum_{k=1}^{N-3} 720\left[\frac{(k)(k+1)(k+2)}{6}\right]\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4 + 3(x_0)^2 + 5\cdots\cdots(m4\omega)$$

Thus, by introducing values for *x*, we get:

	A	B	C	D
10	x_0=	**1**		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
100021			**SUMMATION**	**5.9994**
100022		**(+)**	**$3(x_0)^2$**	3
100023		**(+)**	**5**	5
100024		**(=)**	**$f(x_0)$**	**13.99940**
100025				

If:

	A	B	C	D
10	x_0=	**2**		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
100021			**SUMMATION**	**191.9808**
100022		**(+)**	**$3(x_0)^2$**	12
100023		**(+)**	**5**	5
100024		**(=)**	**$f(x_0)$**	**208.98080**
100025				

If:

	A	B	C	D
10	x_0=	**3**		
11	N=	100,000		
12	(x_0/N)=	0.00001		
13				
100021			**SUMMATION**	**1,457.8542**
100022		**(+)**	**$3(x_0)^2$**	27
100023		**(+)**	**5**	5
100024		**(=)**	**$f(x_0)$**	**1,489.85421**
100025				

The solution to:

$$\frac{d^4}{dx^4} = 720x$$

For inicial condition (x,y,y',y'',y''')=(1,14; 36,126,360)

Is:

$$\rightarrow \boldsymbol{y = 6x^5 + 3x^2 + 5}$$

If the values of this explicit function are tabulated and compared with those obtained through Excel, the following can be seen:

x	$\boldsymbol{y = 6x^5 + 3x^2 + 5}$	$\boldsymbol{y_{\left(\begin{matrix}Calculated\\ with\\ Excel\end{matrix}\right)}}$	$\boldsymbol{Difference}$
1	14	13.99940	-0.0006
2	209	208.98080	-0.0192
3	1,490	1,489.85421	-0.14579

$$\boldsymbol{f(x_0) \cong \left\{\left\{\sum_{k=1}^{N-3}\left[\frac{(k)(k+1)(k+2)}{6}\right].f\left[x_0 - \left(\frac{x_0}{N}\right).(k+3)\right]\right\}.\left(\frac{x_0}{N}\right)^4\right\} + 3(x_0)^2 + 5 \cdots\cdots (m4\omega)}$$

As can be seen, these ***negative differences*** indicate that ***N=100,000*** is a good approximation to infinity (∞), but if an ***N=1,000,000 or larger were chosen***, these ***differences*** would tend towards zero.

Therefore, this method is a way to **calculate antiderivatives** numerically using Excel.

DEFINITE INTEGRALS

SOLVING DEFINITE INTEGRALS WITH EXCEL

To find a ***Definite Integral*** from an **Indefinite Integral**, the following definition is used:

	$\left\{\lim_{n\to\infty}\left\{\left\{\sum_{k=1}^{n}\left[f\left(b-\frac{bk}{n}\right)\right]\right\}\times\left(\frac{b}{n}\right)\right\}\right\}$
(-)	$\left\{\lim_{n\to\infty}\left\{\left\{\sum_{k=1}^{n}\left[f\left(a-\frac{ak}{n}\right)\right]\right\}\times\left(\frac{a}{n}\right)\right\}\right\}$
(=)	$\int_{a}^{b} f(x)\,dx$

In case of Excel, an appoximation to the Definite Integral will be with ***a very large N that tends to infinity,*** which can be with ***N=100,000 or larger***, so the previous definition becomes the following:

	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(b-\frac{bk}{N}\right)\right]\right\}\times\left(\frac{b}{N}\right)\right\}$	
$(-)$	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(a-\frac{ak}{N}\right)\right]\right\}\times\left(\frac{a}{N}\right)\right\}$	$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\psi)$
$(\cong)$	$\int_{a}^{b} f(x)\,dx$	

Note that the definition above that is being subtracted corresponds to the ***First Indefinite Integral*** [which is obtained by substituting **δ=-1** in **Definition (1.1)**].

EXAMPLE:

Find:

$$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}\,dx$$

Using a tendency towards infinity of ***N=100,000***

Sol…

Applying the ***Definition (ψ)***:

	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(b-\frac{bk}{N}\right)\right]\right\}\times\left(\frac{b}{N}\right)\right\}$
$(-)$	$\left\{\left\{\sum_{k=1}^{N}\left[f\left(a-\frac{ak}{N}\right)\right]\right\}\times\left(\frac{a}{N}\right)\right\}$
$(\cong)$	$\int_{a}^{b} f(x)\,dx$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(\psi)$

Hence:

$$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}\,dx$$

Becomes:

	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$
$(-)$	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$
$(\cong)$	$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}\,dx$

NOTE: We will do this as when Equation (M1) was used to calculate Antiderivatives.

1) We enter the Definite Integral in **cell A1**:
2) The following is written in **cell A10** and **cell E10**:
3) We write in **cell A20**:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	=(B10)/(B11)			(a/N)=	=(F10)/(F11)		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21								
22								
23								

According:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

4) We write in **cell A21** and **cell E21**:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21								
22								
23								

=SEQUENCE(100000,1,1,1)

This means there will be **100,000 Rows**
In **1 Column**
Starting **from 1**
Increasing by 1 (from 1 until reaching N=100,000)

5) Therefore, it will be as follows:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1				1			
22	2				2			
23	3				3			
24	4				4			
25	5				5			
100019	9,9999				9,9999			
100020	100,000				100,000			

6) Now in **cell B21** and **cell E21,** we write the following (after autofilling up to ***k=100,000***):

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1				1			
22	2				2			
23	3				3			
24	4				4			
25	5				5			
100019	9,9999				9,9999			
100020	100,000				100,000			

=(B10)-(B12)*(A21)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

=(F10)-(F12)*(E21)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

7) It will look like this:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999			1	-0.99999		
22	2	0.99998			2	-0.99998		
23	3	0.99997			3	-0.99997		
24	4	0.99996			4	-0.99996		
25	5	0.99995			5	-0.99995		
100019	9,9999	0.00001			9,9999	-0.00001		
100020	100,000	0.00000			100,000	0.00000		

8) In **cell C21** and **cell G21**, we write the following (after autofilling up to ***k=100,000***):

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999			1	-0.99999		
22	2	0.99998			2	-0.99998		
23	3	0.99997			3	-0.99997		
24	4	0.99996			4	-0.99996		
25	5	0.99995			5	-0.99995		
100019	9,9999	0.00001			9,9999	-0.00001		
100020	100,000	0.00000			100,000	0.00000		

=exp(-(1/2)*(B21)^2)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

=exp(-(1/2)*(F21)^2)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

9) It will look like this:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654		1	-0.99999	0.60654	
22	2	0.99998	0.60654		2	-0.99998	0.60654	
23	3	0.99997	0.60655		3	-0.99997	0.60655	
24	4	0.99996	0.60655		4	-0.99996	0.60655	
25	5	0.99995	0.60656		5	-0.99995	0.60656	
100019	9,9999	0.00001	1.00000		9,9999	-0.00001	1.00000	
100020	100,000	0.00000	1.00000		100,000	0.00000	1.00000	

10) In **cell D21** and **cell H21**, we write the following (after autofilling up to ***k=100,000***)::

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654		1	-0.99999	0.60654	
22	2	0.99998	0.60654		2	-0.99998	0.60654	
23	3	0.99997	0.60655		3	-0.99997	0.60655	
24	4	0.99996	0.60655		4	-0.99996	0.60655	
25	5	0.99995	0.60656		5	-0.99995	0.60656	
100019	9,9999	0.00001	1.00000		9,9999	-0.00001	1.00000	
100020	100,000	0.00000	1.00000		100,000	0.00000	1.00000	

=(C21)*(B12)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

Equivalent to:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}$$

=(G21)*(F12)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

Equivalent to:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

11) It will look like this:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021								

12) In **cell C100021** and **G100021** write **"SUM B"** and **"SUM A"**, respectively:

13) In **cell D100021** and **cell H100021**, we write:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**				**SUM A**	

=SUM(D21:D100020)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$$

Equivalent to:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}$$

=SUM(H21:H100020)

Represents in Red:

$$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$$

Equivalent to:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

14) It will look like this:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023								
100024								

15) Now, in **cell C100023** and **cell G100023**, we write (due to limited space) the words **0.3XSUM B** and **0.3XSUM A**, respectively.

16) While in **cell D100023** and **cell H100023** we write the following:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**				**0.3XSUM A**	
100024								

=0.398942*(D100021)

Represents in Red:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}$$

=0.398942*(H100021)

Represents in Red:

$$0.398942\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

17) It will look like this:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
6								
7								
8								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
14								
15								
16								
17								
18								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**	**0.34135**			**0.3XSUM A**	**-0.34135**
100024								

18) Now in **cell C100024** we write the word "**DIFFERENCE**".

19) While in **cell D100024** we write:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**	**0.34135**			**0.3XSUM A**	**-0.34135**
100024			**DIFFERENCE**					

=D100023-H100023

Represents in Red:

$$\mathbf{0.398942}\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)\right\}-\mathbf{0.398942}\sum_{k=1}^{100,000}\left\{e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)\right\}$$

According:

	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\right\}\times\left(\frac{1}{100,000}\right)\right\}$
(−)	$0.398942\left\{\left\{\sum_{k=1}^{100,000} e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\right\}\times\left(\frac{-1}{100,000}\right)\right\}$
(≅)	$0.398942\int_{a=-1}^{b=1} e^{-\frac{1}{2}x^2}dx$

20) **Finally**, it ends up like this:

	A	B	C	D	E	F	G	H
1	$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx$							
2								
3								
4								
5								
9								
10	x_0=b=	1			x_0=a=	-1		
11	N=	100,000			N=	100,000		
12	(b/N)=	0.00001			(a/N)=	-0.00001		
13								
19								
20	k	$\left[1-\left(\frac{1}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(1-\frac{1k}{100,000}\right)^2}\times\left(\frac{1}{100,000}\right)$	k	$\left[-1-\left(\frac{(-1)}{100,000}\right)k\right]$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}$	$e^{-\frac{1}{2}\left(-1-\frac{(-1)k}{100,000}\right)^2}\times\left(\frac{-1}{100,000}\right)$
21	1	0.99999	0.60654	**0.00001**	1	-0.99999	0.60654	**-0.00001**
22	2	0.99998	0.60654	**0.00001**	2	-0.99998	0.60654	**-0.00001**
23	3	0.99997	0.60655	**0.00001**	3	-0.99997	0.60655	**-0.00001**
24	4	0.99996	0.60655	**0.00001**	4	-0.99996	0.60655	**-0.00001**
25	5	0.99995	0.60656	**0.00001**	5	-0.99995	0.60656	**-0.00001**
100019	9,9999	0.00001	1.00000	**0.00001**	9,9999	-0.00001	1.00000	**-0.00001**
100020	100,000	0.00000	1.00000	**0.00001**	100,000	0.00000	1.00000	**-0.00001**
100021			**SUM B**	**0.85563**			**SUM A**	**-0.85563**
100022								
100023			**0.3XSUM B**	**0.34135**			**0.3XSUM A**	**-0.34135**
100024			**DIFFERENCE**	**0.68269**				

We can verify that:

$$0.398942\int_{-1}^{1} e^{-\frac{1}{2}x^2}dx \cong 0.682689492$$

¡Therefore, the Excel result of 0.68269 is a good approximation for N=100,000!

BIBLIOGRAPHIC REFERENCES

- For the link to the Book "***On Indefinite Integral***" (e-book on archive.org):

- For the link to the Manual for calculating ***Antiderivatives*** and ***Definite Integrals*** in Spanish (e-book on archive.org):

- To see the complete proof of the origin of the ***First Indefinite Integral*** (video on Youtube):